Inverse bremsstrahlung and temperature relaxation in moderately coupled two-temperature plasmas

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The balance equation for the energy in moderately coupled two-temperature plasmas, in the presence of an external radiation field, is derived and analyzed. The analysis is based on the Singwi-Tosi-Land-Sjolander closure assumption. The different terms in the derived equation are identified as the rate of collisional energy absorption from the external field (inverse bremsstrahlung), and the rate of energy transfer between the electrons and the ions in the presence of the radiation field (relaxation). It is shown how these terms, which have a structurally similar appearance, reduce to known expressions for relaxation and inverse bremsstrahlung in the appropriate limits. It is found that, relative to the known expressions, electron-ion correlation tends to enhance the rates of re1axation and of the inverse bremsstrahlung process.

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I. INTRODUCTION

The theoretical framework for the analysis of strongly coupled plasmas is not yet fully developed. In particular it is not clear what is the minimum approximation scheme (beyond the random phase and Born approximations) which is sufficient for the description of the effect of strong ion-ion and electron-ion coupling on relaxation and transport processes in the plasma, even without an external radiation field [1,2]. Some understanding of this problem may be obtained by considering mild cases where the ion-electron coupling is not so strong.

When the typical relaxation times of electrons and ions are much shorter than typical electron-ion relaxation times [3], one may assume that each specie is close to thermal equilibrium, with different temperatures for the electrons and the ions. In this case, one may proceed along the line of the Singwi-Tosi-Land-Sjolander (STLS) approach for one- [4] and multiple- [5] component plasmas. This approach truncates the Born-Bogoliubov-Green-Kirkwood-Yvon (BBGKY) hierarchy of equations [6,7] for the multiple-particle distribution functions $f(\vec{r},\vec{p},t)$, $f(\vec{r},\vec{p},\vec{r}',\vec{p}',t),\ldots$ using the ansatz $f(\vec{r},\vec{p},\vec{r}',\vec{p}',t)=f(\vec{r},\vec{p},t)f(\vec{r}',\vec{p}',t)g(|\vec{r}-\vec{r}'|)$ (where g is the equilibrium pair distribution function) and then expands the kinetic equation around uniform thermal equilibrium $f^0(\vec{p})$. The result is that the linear response functions are changed from the functions in the absence of correlation [i.e., with $f(\vec{r},\vec{p},\vec{r}',\vec{p}',t)=f(\vec{r},\vec{p},t)f(\vec{r}',\vec{p}',t)$] by a local field correction [8], which is a functional of the pair distribution function $g(|\vec{r}-\vec{r}'|)$. The fluctuation-dissipation theorem supplies another relation between the response functions and the pair correlation function, leading to a closed set of equations for the self-consistent evaluation of the linear response functions and the pair distribution function. This approach was successful in a variety of applications, e.g., in describing the dielectric function of the interacting electron liquid and of a two-component electron-hole liquid, and in getting excellent agreement of the ground state energy with numerical experiments over the whole range of metallic densities (see, for example, Ref. [9]).

In the present work we use the STLS approach to investigate the energy transfer processes in moderately coupled plasmas, close to quasithermal equilibrium with different temperatures for the electrons and the ions. The treatment includes an oscillating electric field which represents a long-wavelength radiation field. The balance equation for the kinetic energy is derived from the second velocity moment of the lowest order equation in the quantum BBGKY hierarchy of equations. The derived equation includes a term which is identified as the rate of collisional energy absorption from the external field (inverse bremsstrahlung), and two additional terms which are identified as the rate of energy transfer between the electrons and ions with or without the radiation field. This is different from existing works (with weak or strong coupling) which specialize either to the inverse bremsstrahlung process [10–21] (by considering, right from the beginning, only the case of infinite ion-to-electron mass ratio) or to temperature relaxation [1,22–24] (by considering systems without an external field). In the present derivation, it is found that both inverse bremsstrahlung and relaxation rates may be written as overlap integrals of the electron and ion spectra in $\vec{k},\omega$ space. This corresponds to a second-order perturbation expansion in the electron-ion interaction potential, however with the interaction "dressed" by the local field correction.

It is found that, in this scheme, the correlation induces two types of corrections in the formulas for the rates of energy transfer and inverse bremsstrahlung: the electron-electron and ion-ion dynamic local field correction which dresses the electron's and ion's response functions [4,5], and a static local field correction which dresses the electron-ion coupling potential.

The standard formulas for the process of temperature relaxation [22,23] (without a radiation field) and collisional absorption [11,10] from the radiation field, as well as some more recent results [1,16,17,21,24] are obtained by taking the proper limits and utilizing the properties of electron and ion spectra to perform the ω integration.

The plan of the paper is as follows: In Sec. II the basic equations are presented and the balance equation for the kinetic energy is derived. In Sec. III, we use the quasilinear approximation to write the rate of energy transfer from radiation to the electrons and ions and between the electrons and ions, in the balance equation, in terms of overlap integrals of the electron and ion spectra in $\vec{k},\omega$ space. The term responsible for the inverse bremsstrahlung process is analyzed in Sec. IV, and Sec. V analyzes the terms responsible for energy transfer between the electrons and ions in the absence of external radiation. An evaluation of the correction factor due to electron-ion correlations is given in Sec. VI, and Sec. VII is devoted to a summary and discussion.

II. THE KINETIC AND TRANSPORT EQUATIONS

In this section, we will develop an expression for the power P^α absorbed by the electrons or the ions in a plasma, by applying the STLS closure assumption to the basic equations of motion. The expression obtained will be bilinear in the particle densities, and will thus allow the introduction of density correlators in the following section. An external oscillating electric field driving the plasma will be included, representing an applied laser. Using Wigner functions will allow us to present quantum-mechanical results and classical results simultaneously.

The N-body Schrödinger equation for a plasma of interacting particles—N^e electrons and N^i ions in a volume V—may be transformed [6,7] to a hierarchy of equations for the Wigner functions $F^\alpha(\vec{r},\vec{p},t)$, $F^{\alpha\beta}(\vec{r},\vec{p},\vec{r}\,',\vec{p}\,',t),\ldots$ where α,β denote either electrons or ions. It is convenient here to modify the normalization of the Wigner functions used in Ref. [6] by defining $f^\alpha(\vec{r},\vec{p},t)=N^\alpha(2\pi\hbar)^{-3}F^\alpha(\vec{r},\vec{p},t)$, $f^{\alpha\beta}(\vec{r},\vec{p},\vec{r}\,',\vec{p}\,',t)=N^\alpha N^\beta(2\pi\hbar)^{-6}F^{\alpha\beta}(\vec{r},\vec{p},\vec{r}\,',\vec{p}\,',t),\ldots$ Many of the equations below are then unchanged in the classical limit $\hbar\to 0$ and can be understood as representing both the quantum case [where $f^\alpha(\vec{r},\vec{p},t)$ are the Wigner functions] and the classical case, with $f^\alpha(\vec{r},\vec{p},t)$ denoting phase-space distribution functions. The expectation values of the particle density, charge current, and kinetic energy at a point $\vec{r}$ and time t are thus

$$n^\alpha(\vec{r},t)=\langle n^\alpha\rangle=\int f^\alpha(\vec{r},\vec{p},t)d^3\vec{p},$$

$$\vec{j}^\alpha(\vec{r},t)=\left\langle q^\alpha n^\alpha\frac{\vec{p}}{m^\alpha}\right\rangle=q^\alpha\int\frac{\vec{p}}{m^\alpha}f^\alpha(\vec{r},\vec{p},t)d^3\vec{p},$$

$$k^\alpha(\vec{r},t)=\left\langle n^\alpha\frac{p^2}{2m^\alpha}\right\rangle=\int\frac{p^2}{2m^\alpha}f^\alpha(\vec{r},\vec{p},t)d^3\vec{p}. \tag{1}$$

The time evolution of $\vec{j}^\alpha(\vec{r},t),k^\alpha(\vec{r},t)$ is affected by correlations. In order to examine this effect, we shall apply one of the simplest known approaches which goes beyond the Hartree (random phase) approximation: the hierarchy of equations which connects the single-particle distributions to the multiparticle distributions is truncated by assuming [4,5,25]

$$f^{\alpha\beta}(\vec{r},\vec{p},\vec{r}\,',\vec{p}\,',t)=f^\alpha(\vec{r},\vec{p},t)f^\beta(\vec{r}\,',\vec{p}\,',t)g^{\alpha\beta}(\vec{r}-\vec{r}\,'). \tag{2}$$

Assumption (2) with the identification of g with the pair distribution function in the equilibrium state [4,25] reduces the hierarchy of equations to a set of coupled kinetic equations for the single-particle Wigner functions [25]. In the classical case one obtains the kinetic equation

$$\frac{\partial}{\partial t}f^\alpha(\vec{r},\vec{p},t)+\frac{\vec{p}}{m^\alpha}\cdot\frac{\partial}{\partial\vec{r}}f^\alpha(\vec{r},\vec{p},t)+\frac{\partial}{\partial\vec{p}}f^\alpha(\vec{r},\vec{p},t)\cdot m^\alpha\vec{a}^\alpha=0, \tag{3}$$

where the accelerations are given by

$$m^\alpha\vec{a}^\alpha=-\frac{\partial}{\partial\vec{r}}q^\alpha\phi^{\rm ext}(\vec{r},\vec{t})-\int\left[n^\alpha(\vec{r}\,',t)g^{\alpha\alpha}(\vec{r}-\vec{r}\,')\frac{\partial}{\partial\vec{r}}V^{\alpha\alpha}(\vec{r}-\vec{r}\,')+n^\gamma(\vec{r}\,',t)g^{\alpha\gamma}(\vec{r}-\vec{r}\,')\frac{\partial}{\partial\vec{r}}V^{\alpha\gamma}(\vec{r}-\vec{r}\,')\right]d^3\vec{r}\,', \tag{4}$$

with $\alpha=e$, i and $\gamma=i$, e, respectively, and $V^{\alpha\beta}(\vec{r}-\vec{r}\,')$ denoting the different interparticle interaction potentials (β is equal to either α or γ). The quantum-mechanical form of the kinetic equation is obtained by applying the Fourier transform

$$\frac{\partial}{\partial\vec{p}}f^\alpha(\vec{r},\vec{p},t)=\int e^{i\vec{\xi}\cdot(\vec{\eta}-\vec{p})}(-i\vec{\xi})f^\alpha(\vec{r},\vec{\eta},t)\frac{d^3\vec{\xi}d^3\vec{\eta}}{(2\pi)^3} \tag{5}$$

and making the replacements

$$\vec{\xi}\cdot\frac{\partial}{\partial\vec{r}}\phi^{\rm ext}(\vec{r},t)\Rightarrow\frac{1}{\hbar}\left[\phi^{\rm ext}\left(\vec{r}+\frac{1}{2}\hbar\vec{\xi},t\right)-\phi^{\rm ext}\left(\vec{r}-\frac{1}{2}\hbar\vec{\xi},t\right)\right],$$

$$\vec{\xi}\cdot\frac{\partial}{\partial\vec{r}}V^{\alpha\beta}(\vec{r}-\vec{r}\,')\Rightarrow\frac{1}{\hbar}\left[V^{\alpha\beta}\left(\vec{r}+\frac{1}{2}\hbar\vec{\xi}-\vec{r}\,'\right)-V^{\alpha\beta}\left(\vec{r}-\frac{1}{2}\hbar\vec{\xi}-\vec{r}\,'\right)\right] \tag{6}$$

in Eqs. (3) and (4), where it is clear that taking $\hbar\to 0$ reproduces the classical results.

The above approach of displaying the more transparent classical results, and then stating whenever necessary the replacement needed in order to generate the quantum-mechanical results, will be used throughout the rest of this paper.

The Hartree approximation is recovered here by ignoring correlations (i.e., taking $g^{\alpha\beta}=1$), which gives the Vlasov equation for a two-component plasma [26] (the extension to a many-component plasma, containing several species of ions, is obvious). The present notation allows us to consider both Coulomb interactions with $V^{\alpha\beta}(\vec{r}-\vec{r}\,')=q^\alpha q^\beta/|\vec{r}-\vec{r}\,'|$, and partially ionized plasmas with electron-ion interactions represented by pseudopotentials and ion-ion interactions affected by the repulsive cores.

Taking the first and second velocity moments of Eq. (3), and averaging over the volume V one gets the following equations for the momentum and energy:

$$\frac{\partial}{\partial t}\vec{J}^{\alpha}(t) = q^{\alpha}\int \frac{\vec{p}}{m^{\alpha}}\frac{\partial}{\partial t}f^{\alpha}(\vec{r},\vec{p},t)d^3\vec{p}\frac{d^3\vec{r}}{V} = \frac{q^{\alpha}}{m^{\alpha}}\int \vec{E}^{\text{ext}}(\vec{r},t)q^{\alpha}n^{\alpha}(\vec{r},t)\frac{d^3\vec{r}}{V} + \frac{q^{\alpha}}{m^{\alpha}}\int \vec{E}^{\text{eff},\alpha}(\vec{r},t)q^{\alpha}n^{\alpha}(\vec{r},t)\frac{d^3\vec{r}}{V} \tag{7}$$

and

$$\frac{\partial}{\partial t}K^{\alpha}(t) = \frac{\partial}{\partial t}\left(\int \frac{p^2}{2m^{\alpha}}f^{\alpha}(\vec{r},\vec{p},t)d^3\vec{p}\frac{d^3\vec{r}}{V}\right) = \int [\vec{E}^{\text{ext}}(\vec{r},t) + \vec{E}^{\text{eff},\alpha}(\vec{r},t)]\cdot\vec{j}^{\alpha}(\vec{r},t)\frac{d^3\vec{r}}{V}. \tag{8}$$

Here $\vec{E}^{\text{eff},\alpha}(\vec{r},t)$ is the effective field,

$$\vec{E}^{\text{eff},\alpha}(\vec{r},t) = -\vec{\nabla}\phi^{\text{eff},\alpha}(\vec{r},t), \tag{9}$$

where the effective potential $\phi^{\text{eff},\alpha}(\vec{r},t)$ can be written as

$$q^{\alpha}\phi^{\text{eff},\alpha}(\vec{r},t) = \int \{\Psi^{\alpha\alpha}(\vec{r}-\vec{r}')n^{\alpha}(\vec{r}',t) + \Psi^{\alpha\gamma}(\vec{r}-\vec{r}')n^{\gamma}(\vec{r}',t)\}d^3\vec{r}' \tag{10}$$

in terms of an effective pair interaction $\Psi^{\alpha\beta}(\vec{r})$ defined by

$$\vec{\nabla}\Psi^{\alpha\beta}(\vec{r}) = g^{\alpha\beta}(\vec{r})\vec{\nabla}V^{\alpha\beta}(\vec{r}). \tag{11}$$

Equations (7) and (8) apply both to the classical and quantum-mechanical cases. Formally, the macroscopic equation (7) looks like Newton's equation for the fluid velocity $\vec{U}^{\alpha}$, i.e., $(\partial/\partial t)\vec{U}^{\alpha}(\vec{r},t)=(q^{\alpha}/m^{\alpha})\vec{E}^{\text{eff},\alpha}(\vec{r},t)$, and Eq. (8) tells us that the change in the energy of the α fluid equals to the work performed by the external field and the effective field on the α specie (i.e., the ohmic dissipation).

It is convenient at this point to specialize to the case of a uniform monochromatic external field, representing a long-wavelength laser: $\vec{E}^{\text{ext}}(\vec{r},t)=\vec{E}_0\cos\omega_0 t$ (with adiabatic switching on from $t\to-\infty$). The integrals involving the external field in Eqs. (7) and (8) can then be preformed, yielding N^{α} and $\vec{J}^{\alpha}$, respectively. Substituting for the latter, transforming to $\vec{k}$ space (with $f_{\vec{k}}=\int e^{-i\vec{k}\cdot\vec{r}}f(\vec{r})d^3\vec{r}$ and $f(\vec{r})=\int e^{i\vec{k}\cdot\vec{r}}f_{\vec{k}}d^3\vec{k}/(2\pi)^3$, and using continuity [i.e., the lowest velocity moment of the kinetic equation (3), $q^{\alpha}(\partial/\partial t)n_{\vec{k}}^{\alpha}(t)+i\vec{k}\cdot\vec{j}_{\vec{k}}^{\alpha}(t)=0$] gives the rate of change of the kinetic energy of the α specie as

$$\frac{\partial}{\partial t}K^{\alpha}(t) = \frac{q^{\alpha 2}}{2\omega_0 m^{\alpha}}\frac{N^{\alpha}}{V}E_0^2\sin(2\omega_0 t) + \frac{q^{\alpha}}{m^{\alpha}}\vec{E}_0\cos(\omega_0 t)\frac{1}{V}\int_{-\infty}^{t}d\tau\int\frac{d^3\vec{k}}{(2\pi)^3}\times(-i\vec{k})\Psi_{\vec{k}}^{\alpha\gamma}n_{\vec{k}}^{\gamma}(\tau)n_{-\vec{k}}^{\alpha}(\tau) - \frac{1}{V}\int\frac{d^3\vec{k}}{(2\pi)^3}\Psi_{\vec{k}}^{\alpha\alpha}n_{\vec{k}}^{\alpha}(t)\frac{\partial}{\partial t}n_{-\vec{k}}^{\alpha}(t) - \frac{1}{V}\int\frac{d^3\vec{k}}{(2\pi)^3}\Psi_{\vec{k}}^{\alpha\gamma}n_{\vec{k}}^{\gamma}(t)\frac{\partial}{\partial t}n_{-\vec{k}}^{\alpha}(t) \tag{12}$$

(the effective pair potential is here $\Psi_{k}^{\alpha\beta}=(\vec{k}/k^2)\int g_{\vec{k}-\vec{k}'}^{\alpha\beta}\vec{k}'V_{\vec{k}'}^{\alpha\beta}d^3\vec{k}'/(2\pi)^3$, and depends only on the magnitude k). The first term on the right-hand side is the direct response of the α specie to the external field, the second term represents absorption from the external field (i.e., the inverse bremsstrahlung process), the third term is the change in potential energy of the α specie, and the last term represents the transport of energy between the α specie and the γ specie (relaxation). Moving the potential energy term to the right-hand side of the equation, using its symmetry one finds that the rate of change of the total energy of the α specie is

$$P^{\alpha} = \frac{\partial}{\partial t}\left(K^{\alpha}(t) + \frac{1}{2V}\int\frac{d^3\vec{k}}{(2\pi)^3}\Psi_{\vec{k}}^{\alpha\alpha}n_{\vec{k}}^{\alpha}(t)n_{-\vec{k}}^{\alpha}(t)\right) = \frac{q^{\alpha 2}}{2\omega_0 m^{\alpha}}\frac{N^{\alpha}}{V}E_0^2\sin(2\omega_0 t) + \frac{q^{\alpha}}{m^{\alpha}}\vec{E}_0\cos(\omega_0 t)\frac{1}{V}\int_{-\infty}^{t}d\tau\int\frac{d^3\vec{k}}{(2\pi)^3}(-i\vec{k})\Psi_{\vec{k}}^{\alpha\gamma}n_{\vec{k}}^{\gamma}(\tau)n_{-\vec{k}}^{\alpha}(\tau) - \frac{1}{V}\int\frac{d^3\vec{k}}{(2\pi)^3}\Psi_{\vec{k}}^{\alpha\gamma}n_{\vec{k}}^{\gamma}(t)\frac{\partial}{\partial t}n_{-\vec{k}}^{\alpha}(t). \tag{13}$$

Note that this expression for the power, which is bilinear in the densities, follows directly from the STLS closure ansatz. No linearization approximation is needed.

III. THE QUASILINEAR APPROXIMATION FOR THE ENERGY EQUATION

In the quasilinear approximation [26], one assumes that the two components of the plasma, electrons, and ions can be considered separately, with their effects on each other treated as a small perturbation around their thermodynamic equilibrium states (Ref. [32] shows that for some purposes this is a good approximation even at the relatively high densities of liquid metals, provided that the electron-ion interaction is represented by an appropriate pseudopotential). Using $\langle\cdots\rangle$ to denote averaging over the thermodynamic ensemble, we will thus approximate the bilinear correlators in Eq. (13) by expressing them in terms of linear susceptibilities. For example,

$$\langle n^{\alpha}_{\vec{k}}(\tau) n^{\gamma}_{-\vec{k}}(\tau)\rangle = \langle n^{\alpha}_{\vec{k}}(\tau) n^{\gamma,\mathrm{ind}}_{-\vec{k}}(\tau)\rangle + \langle n^{\alpha,\mathrm{ind}}_{\vec{k}}(\tau) n^{\gamma}_{-\vec{k}}(\tau)\rangle, \quad (14)$$

where the induced perturbation $n^{\mathrm{ind},\alpha}_{\vec{k}}(\tau)$ and the inducing perturbation $n^{\gamma}_{\vec{k}}$ are linearly related to each other (at zeroth order, the fluctuations of the α specie and the γ specie are of course uncorrelated).

The specific form of the linear relation between the induced and inducing perturbations may be obtained from the kinetic equation (3). The solution of this equation becomes simpler when transformed into a frame in which the effect of the radiation field on one of the components, say α, is eliminated [11,12], as will be done explicitly below. Let us then display first the expressions for the susceptibilities in the absence of an external field. Linearization of Eq. (3), assuming a small perturbation around thermal equilibrium with momentum distribution $f^{0\alpha}(\vec{p})$, leads then in the classical case to the relation

$$\begin{aligned} n^{\alpha,\mathrm{ind}}_{\vec{k},\omega} &= \int f^{\alpha,\mathrm{ind}}_{\vec{k},\omega}(\vec{p}) d^3\vec{p} \\ &= n^{\alpha,\mathrm{ind}}_{\vec{k},\omega} \int g^{\alpha\alpha}_{\vec{k}-\vec{k}'} V^{\alpha\alpha}_{\vec{k}'} \vec{k}' \left(\int \frac{\partial f^{0\alpha}}{\partial \vec{p}} \frac{d^3\vec{p}}{\omega - \vec{k}\cdot\frac{\vec{p}}{m^{\alpha}}} \right) \frac{d^3\vec{k}'}{(2\pi)^3} \\ &\quad + n^{\gamma}_{\vec{k},\omega} \int g^{\alpha\gamma}_{\vec{k}-\vec{k}'} V^{\alpha\gamma}_{\vec{k}'} \vec{k}' \left(\int \frac{\partial f^{0\alpha}}{\partial \vec{p}} \frac{d^3\vec{p}}{\omega - \vec{k}\cdot\frac{\vec{p}}{m^{\alpha}}} \right) \frac{d^3\vec{k}'}{(2\pi)^3}, \end{aligned} \quad (15)$$

where here the replacement $\vec{k}'\cdot\partial f^{0\alpha}/\partial\vec{p} \Rightarrow (1/\hbar) \times \left[f^{0\alpha}\left(\vec{p}+\tfrac{1}{2}\hbar\vec{k}'\right) - f^{0\alpha}\left(\vec{p}-\tfrac{1}{2}\hbar\vec{k}'\right)\right]$ gives the quantum-mechanical result.

Following Ref. [25] (see also Ref. [28]) we define the response function

$$\chi^{0\alpha}(\vec{k},\vec{k}',\omega) = -\vec{k}' \int \frac{\partial f^{0\alpha}}{\partial \vec{p}} \frac{d^3\vec{p}}{\omega - \vec{k}\cdot\frac{\vec{p}}{m^{\alpha}}} \quad (16)$$

and a dynamic effective interaction

$$\Phi^{\alpha\beta}(\vec{k},\omega) = \frac{1}{\chi^{0\alpha}(\vec{k},\omega)} \int g^{\alpha\beta}_{\vec{k}-\vec{k}'} V^{\alpha\beta}_{\vec{k}'} \chi^{0\alpha}(\vec{k},\vec{k}',\omega) \frac{d^3\vec{k}'}{(2\pi)^3}, \quad (17)$$

where $\chi^{0\alpha}(\vec{k},\omega) = \chi^{0\alpha}(\vec{k},\vec{k},\omega)$ (these susceptibilities are complex functions, as ω is understood to have a positive infinitesimal imaginary part representing the adiabatic switching on; thus $\chi^{0\alpha}$ is replaced by its complex conjugate upon reversal of the sign of Reω; this property is shared by $\Phi^{\alpha\beta}$, and also by χ^{α} defined next). With these definitions, the density-density linear response relation reads

$$n^{\alpha,\mathrm{ind}}_{\vec{k},\omega} = \chi^{\alpha}(\vec{k},\omega) V^{\alpha\gamma}_{\vec{k}} n^{\gamma}_{\vec{k},\omega} \quad (18)$$

with the density response function

$$\chi^{\alpha}(\vec{k},\omega) = \left(\frac{\Phi^{\alpha\gamma}(\vec{k},\omega)}{V^{\alpha\gamma}_{\vec{k}}} \right) \frac{\chi^{0\alpha}(\vec{k},\omega)}{1 - \Phi^{\alpha\alpha}(\vec{k},\omega)\chi^{0\alpha}(\vec{k},\omega)}, \quad (19)$$

and so the corrections due to the nontrivial correlation function $g^{\alpha\gamma}$ appear only in the factor in parentheses (the effects of $g^{\alpha\alpha}$ are represented by the replacement of $V^{\alpha\alpha}$ by $\Phi^{\alpha\alpha}$). Note that the response function relevant to the evaluation of quasilinear relations such as Eq. (14) is the density-density response function in a *one-component plasma*, viewing the fluctuations in the other component as the perturbing quantity.

In the presence of a uniform external field $\vec{E}^{\mathrm{ext}} = \vec{E}_0 \cos \omega_0 t$, one may transform to a rotated phase space $\vec{r} \to \vec{\rho}^{\alpha}(\vec{r},t) = \vec{r} + \vec{\varepsilon}^{\alpha} \cos \omega_0 t$ and $\vec{v} \to \vec{u}^{\alpha}(\vec{r},t) = \vec{v} - \omega_0 \vec{\varepsilon}^{\alpha} \sin \omega_0 t$, where $\vec{\varepsilon}^{\alpha} = (q^{\alpha}/m^{\alpha}\omega_0^2)\vec{E}_0$, which eliminates the field from Eq. (3). In this "α rotated frame" the relation (18) holds. The quantities evaluated in this frame will be denoted by; α. For example, $n^{\alpha}(\vec{r},t;\alpha)$ is the density of the α component in the α rotated frame. Any scalar quantity $a(\vec{r},t)$ solved in the rotated frame and transformed to the laboratory frame is $a(\vec{r},t) = a(\vec{r}+\vec{\varepsilon}^{\alpha}\cos \omega_0 t, t; \alpha)$. In $\vec{k}$ space the transformation reads $a_{\vec{k}}(t) = a_{\vec{k}}(t;\alpha) e^{i\vec{k}\cdot\vec{\varepsilon}^{\alpha}\cos \omega_0 t} = a_{\vec{k}}(t;\alpha)\sum_{n=-\infty}^{\infty} i^n J_n(\vec{k}\cdot\vec{\varepsilon}^{\alpha}) e^{in\omega_0 t}$, where we have used the expansion [27] $e^{iz\cos\theta} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{in\theta}$ where J_n is the Bessel function. Similar expressions allow us to transform to and from the γ rotated frame. For example, for one of the density correlators we obtain

$$\begin{aligned} \overline{\frac{1}{V} n^{\gamma,\mathrm{ind}}_{\vec{k}}(t) n^{\alpha}_{-\vec{k}}(t)} &= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \chi^{\gamma}(\vec{k},\omega) V^{\alpha\gamma}_{\vec{k}} \\ &\quad \times \overline{\int dt'\, e^{-i\omega(t-t')} e^{i\vec{k}\cdot\vec{\varepsilon}(\cos \omega_0 t - \cos \omega_0 t')} \frac{1}{V} n^{\alpha}_{\vec{k}}(t';\alpha) n^{\alpha}_{-\vec{k}}(t;\alpha)} \\ &= \sum_{m=-\infty}^{\infty} [J_m(\vec{k}\cdot\vec{\varepsilon})]^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \chi^{\gamma}(\vec{k},\omega) V^{\alpha\gamma}_{\vec{k}} S^{\alpha}(\vec{k},\omega - m\omega_0), \end{aligned} \quad (20)$$

where $\vec{\varepsilon} = \vec{\varepsilon}^{\gamma} - \vec{\varepsilon}^{\alpha}$, the overbar denotes both ensemble averaging and averaging over a period of the external field, $\overline{(\cdots)} = \int_0^{2\pi/\omega_0} \langle\cdots\rangle(\omega_0 dt/2\pi)$, and we have used the time-translation invariance of the fluctuations in the thermal ensemble, $\langle n^{\alpha}_{\vec{k}}(t';\alpha) n^{\alpha}_{-\vec{k}}(t;\alpha)\rangle = \langle n^{\alpha}_{\vec{k}}(0;\alpha) n^{\alpha}_{-\vec{k}}(t-t';\alpha)\rangle$, and the definition

$$S^{\alpha}(\vec{k},\omega) = \int dt'' e^{-i\omega t''} \frac{1}{V} \langle n^{\alpha}_{\vec{k}}(0;\alpha) n^{\alpha}_{-\vec{k}}(t'';\alpha)\rangle \quad (21)$$

for the dynamic structure factor (which is real and even in ω).

Application of this approach to the averaged power of Eq. (13), in the quasilinear approximation (14), leads to

$$\wp^{\alpha} = \overline{P^{\alpha}} = \wp^{E\alpha} + \wp^{\alpha\gamma} - \wp^{\gamma\alpha}, \tag{22}$$

where the direct reactive term vanishes upon averaging, and the remaining terms are identified as $\wp^{E\alpha}$, the heating of the α specie by the external radiation, $\wp^{\alpha\gamma}$, the heating of the α specie by thermal fluctuations in the γ specie, and $\wp^{\gamma\alpha}$, the cooling of the α specie by transfer of energy to the γ specie. The power absorbed from the radiation field can be rewritten as

$$\wp^{E\alpha} = -\frac{\frac{q^\alpha}{m^\alpha}}{\frac{q^\alpha}{m^\alpha} - \frac{q^\gamma}{m^\gamma}} \int \frac{d^3\vec{k}}{(2\pi)^3} V_{\vec{k}}^{\alpha\gamma} \Psi_{\vec{k}}^{\alpha\gamma} \sum_{m=-\infty}^{\infty} [J_m(\vec{k}\cdot\vec{\varepsilon})]^2 \times \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} m\omega_0 \{\mathrm{Im}[\chi^\alpha(\vec{k},\omega)] S^\gamma(\vec{k},\omega - m\omega_0) - \mathrm{Im}[\chi^\gamma(\vec{k},\omega)] S^\alpha(\vec{k},\omega - m\omega_0), \tag{23}$$

by using the relation

$$\int_0^{2\pi/\omega_0} \frac{\omega_0 dt}{2\pi} \cos\,\omega_0 t \int_{-\infty}^{t} d\tau e^{i\vec{k}\cdot\vec{\varepsilon}\cos\,\omega_0\tau} e^{-i\vec{k}\cdot\vec{\varepsilon}\cos\,\omega_0(\tau - t'')} = \sum_{m=-\infty}^{\infty} [J_m(\vec{k}\cdot\vec{\varepsilon})]^2 e^{im\omega_0 t''} \frac{m}{\omega_0 \vec{k}\cdot\vec{\varepsilon}}, \tag{24}$$

which follows from the Bessel property $J_{n+1}(x) + J_{n-1}(x) = (2n/x)J_n(x)$ [27] [recall that interchanging α and γ in Eq. (20) leads to a sign reversal in $\vec{\varepsilon}$]. The real part of the susceptibility does not contribute here because of its symmetry in frequency, and because $J_m^2(x)$ is symmetric in m, as well as in x. The power absorbed from the thermal fluctuations of the γ specie may similarly be rewritten as

$$\wp^{\alpha\gamma} = -\int \frac{d^3\vec{k}}{(2\pi)^3} V_{\vec{k}}^{\alpha\gamma} \Psi_{\vec{k}}^{\alpha\gamma} \sum_{m=-\infty}^{\infty} [J_m(\vec{k}\cdot\vec{\varepsilon})]^2 \times \int_{-\infty}^{\infty} \left(\omega - m\omega_0 \frac{\frac{q^\alpha}{m^\alpha}}{\frac{q^\alpha}{m^\alpha} - \frac{q^\gamma}{m^\gamma}} \right) \mathrm{Im}[\chi^\alpha(\vec{k},\omega)] \times S^\gamma(\vec{k},\omega - m\omega_0) \frac{d\omega}{2\pi}, \tag{25}$$

which is obtained using

$$\int_0^{2\pi/\omega_0} \frac{\omega_0 dt}{2\pi} (i\omega + i\omega_0 \vec{k}\cdot\vec{\varepsilon}^{\alpha} \sin\,\omega_0 t) e^{i\vec{k}\cdot\vec{\varepsilon}\cos\,\omega_0 t} e^{-i\vec{k}\cdot\vec{\varepsilon}\cos\,\omega_0 t(t - t'')} = \sum_{m=-\infty}^{\infty} [J_m(\vec{k}\cdot\vec{\varepsilon})]^2 e^{im\omega_0 t''} \left(i\omega - im\omega_0 \frac{\vec{k}\cdot\vec{\varepsilon}^{\alpha}}{\vec{k}\cdot\vec{\varepsilon}} \right) \tag{26}$$

(the factor in parentheses represents the time derivative in $(-\partial/\partial t) n_{\vec{k}}^{\alpha,\mathrm{ind}}$). The last term $\wp^{\gamma\alpha}$ is identical to Eq. (25) with α and γ interchanged, as is physically required [it can also be obtained here by representing the derivative in $(-\partial/\partial t) n_{-\vec{k}}^{\alpha}$ by the factor $-i(\omega - m\omega_0) - i\omega_0 \vec{k}\cdot\vec{\varepsilon}^{\alpha} \sin\,\omega_0 t$].

The fact that this derivation deals with the various energetic processes in the same manner allows one to rewrite the result as

$$\wp^{\alpha} = \wp^{E\alpha} + \wp^{\alpha\gamma} - \wp^{\gamma\alpha} = \int \frac{d^3\vec{k}}{(2\pi)^3} V_{\vec{k}}^{\alpha\gamma} \Psi_{\vec{k}}^{\alpha\gamma} \sum_{m=-\infty}^{\infty} [J_m(\vec{k}\cdot\vec{\varepsilon})]^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \times \{a_m^{E\alpha}(k,\omega) + a_m^{\alpha\gamma}(k,\omega) - a_m^{\gamma\alpha}(k,\omega)\}, \tag{27}$$

where

$$a_m^{E\alpha}(k,\omega) = \frac{\frac{q^\alpha}{m^\alpha}}{\frac{q^\alpha}{m^\alpha} - \frac{q^\gamma}{m^\gamma}} \left(\frac{\pi\omega}{T^\alpha} + \frac{\pi(\omega - m\omega_0)}{T^\gamma} \right) \times m\omega_0 S^\alpha(\vec{k},\omega) S^\gamma(\vec{k},\omega - m\omega_0) \tag{28}$$

and

$$a_m^{\alpha\gamma}(k,\omega) = \frac{\pi\omega}{T^\alpha} \left(\omega - m\omega_0 \frac{\frac{q^\alpha}{m^\alpha}}{\frac{q^\alpha}{m^\alpha} - \frac{q^\gamma}{m^\gamma}} \right) S^\alpha(\vec{k},\omega) S^\gamma(\vec{k},\omega - m\omega_0). \tag{29}$$

In these expressions, the fluctuation-dissipation theorem

$$S^\alpha(\vec{k},\omega) = -\frac{T^\alpha}{\pi\omega} \mathrm{Im}\chi^\alpha(\vec{k},\omega) \tag{30}$$

has been used (see, e.g., Ref. [29]). The factors of $T/\pi\omega$ must be replaced by $(\hbar/2\pi)\coth(\hbar\omega/2T)$ in the quantum-mechanical case [in the second term of Eq. (28) the frequency variable has been shifted and the sign of m reversed].

The energy balance equation (27) [with relations (28) and (29)] is the main result of the present work. It expresses both $\wp^{Ee}$, the energy transferred from radiation to the electrons, and $\wp^{ei} - \wp^{ie}$, the energy transferred between the electrons and ions, in terms of the overlap of electron and ion spectra, $S^e(\vec{k},\omega)$ and $S^i(\vec{k},\omega)$, and the effective interaction $\Psi^{ei}(\vec{k})$ which accounts for electron-ion correlations. The reduction of these expressions to well-known results, in various limits, is presented in the following two sections.

IV. INVERSE BREMSSTRAHLUNG

The power absorbed by the electrons in a plasma from the radiation field, i.e., the power of the inverse bremsstrahlung process, is given by Eq. (28):

$$\wp^{Ee} = \frac{\frac{e}{m^e}}{\frac{e}{m^e} - \frac{Ze}{m^i}} \int \frac{d^3\vec{k}}{(2\pi)^3} V^{ei}_{\vec{k}} \Psi^{ei}_{\vec{k}} \sum_{m=-\infty}^{\infty} [J_m(\vec{k}\cdot\vec{\varepsilon})]^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} m\omega_0 \times S^e(\vec{k},\omega) S^i(\vec{k},\omega - m\omega_0) \left\{ \frac{\pi\omega}{T^e} + \frac{\pi(\omega - m\omega_0)}{T^i} \right\}. \quad (31)$$

The spectrum of ion fluctuations, $S^i(\vec{k},\omega - m\omega_0)$, drops to zero very rapidly as the phase velocity $|\omega - m\omega_0|/k$ of the excitation exceeds the ion thermal velocity $\sqrt{T^i/m^i}$, or as the frequency $|\omega - m\omega_0|$ exceeds the ion plasma frequency $\omega_{pi} = \sqrt{4\pi n^{0i} e^2/m^i}$. In most cases, for the electrons, this is a very low frequency range, i.e., the range which contributes to the ω integration is characterized by $|\omega - m\omega_0| \ll k\sqrt{T^e/m^e}$. Using this feature one may expand χ^e around the frequency $\omega = m\omega_0$ in the ω integral. The result is

$$\int_{-\infty}^{\infty} m\omega_0 S^e(\vec{k},\omega) S^i(\vec{k},\omega - m\omega_0) \left\{ \frac{\pi\omega}{T^e} + \frac{\pi(\omega - m\omega_0)}{T^i} \right\} d\omega$$
$$= m\omega_0 \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\partial^n}{\partial\omega^n} S^e(\vec{k},\omega) \left\{ \frac{\pi\omega}{T^e} + \frac{\pi(\omega - m\omega_0)}{T^i} \right\} \right)_{\omega = m\omega_0} \times \int_{-\infty}^{\infty} d\omega (\omega - m\omega_0)^n S^i(\vec{k},\omega - m\omega_0)$$
$$\simeq m^2\omega_0^2 \frac{\pi}{T^e} S^e(\vec{k}, m\omega_0) n^{0i} S^i(\vec{k}), \quad (32)$$

where we have used the symmetry of $S^i(\vec{k},\omega)$ to cancel all the odd terms in the Taylor expansion, neglected the $n=2$ and higher terms, and used the definition of the static structure factor [Eq. (7.67) in Ref. [29]]:

$$S^i(k) = \frac{1}{n^{0i}} \int S^i(\vec{k},\omega) d\omega. \quad (33)$$

Combining these results, one finds that in the lowest significant order in m^e/m^i we have

$$\wp^{Ee} = -\int \frac{d^3\vec{k}}{(2\pi)^3} V^{ei}_{\vec{k}} \Psi^{ei}_{\vec{k}} \sum_{m=-\infty}^{\infty} [J_m(\vec{k}\cdot\vec{\varepsilon}^e)]^2 m\omega_0 \mathrm{Im}[\chi^e(\vec{k}, m\omega_0)] n^{i0} S^i(\vec{k}). \quad (34)$$

In the absence of correlations and for Coulomb interactions we have $g^{ei}_{\vec{k}-\vec{k}'} = \delta^3(\vec{k}-\vec{k}')$, $\Psi^{ei}_{\vec{k}} \to V^{ei}_{\vec{k}} = -(4\pi Z e^2/k^2)$, $S^i(\vec{k}) \to 1$,

$$\chi^e(\vec{k},\omega) \to \frac{\chi^{0e}(k,\omega)}{1 - \frac{4\pi Z e^2}{k^2}\chi^{0e}(\vec{k},\omega)}.$$

In the classical-mechanics case, Eq. (34) then coincides with the results of Ref. [13] for the energy transmitted from the radiation to the electrons. [see Eq. (19) and (20) therein].

V. RELAXATION IN THE ABSENCE OF RADIATION

The term $\wp^{ei} - \wp^{ie}$ in the energy equation (27) accounts for the exchange of energy between electrons and ions. In the absence of a radiation field, only the $m=0$ term survives and we are left with

$$\wp^e = \int \frac{d^3\vec{k}}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} V^{ei}_{\vec{k}} \Psi^{ei}_{\vec{k}} \omega \left\{ \frac{\pi\omega}{T^e} - \frac{\pi\omega}{T^i} \right\} S^i(\vec{k},\omega) S^e(\vec{k},\omega). \quad (35)$$

For Coulomb interactions in the absence of correlations $\Psi^{ei}(\vec{k}) \to V^{ei}_{\vec{k}} = -4\pi Z e^2/k^2$, Eq. (35) (with the help of the fluctuation-dissipation theorem) reduces to the form of Eq. (36) in Ref. [30], which was shown in Ref. [24] to be equivalent to the well-known results developed by Landau [22] and Spitzer [23].

VI. THE EFFECT OF CORRELATIONS

The above comparisons of results (34) and (35) to the literature exhibit the influence of the effective interaction $\Psi^{ei}(k)$ on energetic processes in the present approach, beyond the effect of the dynamic effective interaction $\Phi^{ei}(k,\omega)$ which appears in the response functions. A numerical estimate of $\Psi^{ei}(k)$ requires knowledge of the electron-ion pair distribution function $g^{ei}(r)$ [which in the STLS approach is obtained self-consistently from Eq. (19), the fluctuation-dissipation theorem (30), and the definition of g in terms of S]. While finding $g^{ei}(r)$ accurately is a formidable task which is well beyond the scope of the present work, a rough estimate of the effects may be obtained by using the linear formula [31,32]

$$g^{ei}(k) = \frac{1}{\sqrt{n^{0e}n^{0i}}} v^{ps}(k)\chi^e(k)S^i(k) + \delta^3(\vec{k}). \quad (36)$$

In the above equation $\chi^e(k) = \chi^e(k,\omega=0)$ (which is real), the static ion structure factor $S^i(k)$ is taken from the solution of the Percus-Yevick equation for a fluid of hard spheres, and the pseudopotential v^{ps} is taken as an empty core potential of radius R_c, $v^{ps} = (-4\pi Z e^2/k^2)\cos(kR_c)$. The static structure factor $S^{ie}(k)$ for Al, Bi, and Mg at normal density and $T^e = 20$ eV, taking the density parameter r_s, empty core radius R_c from the literature, using a packing fraction of $\eta = 0.46$ are shown in Fig. 1. [For comparison and analysis of $S^{ie}(k)$ in these cases see Refs. [31,32].]

The correction factor which replaces the coupling coefficient $(v^{ps})^2$ by $v^{ps}\Psi^{ie}$ is shown in Fig. 2. From this figure it is clear that the correlation effect enhances the coupling coefficient, in the range $k < 1.5\sqrt{8mT^e/\hbar^2}$, which is the relevant range for the integral in Eq. (35) which evaluates the rate of temperature relaxation. This is an indication that in moderately coupled plasmas, the electron-ion correlation enhances the rate of temperature relaxation. Similarly, the rate of absorption via the inverse bremsstrahlung process is enhanced.

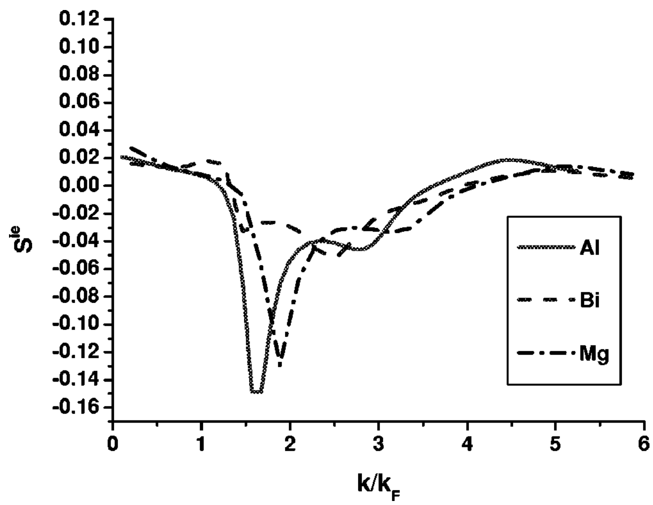

FIG. 1. (Color online) S^{ie} at T_e=20 eV and normal density, for Bi $(Z=5,r_s=2.25,R_c=1.15a_0)$, A1 $(Z=3,r_s=2.07,R_c=1.53a_0)$, Mg$(Z=2,r_s=2.66,R_c=1.31a_0)$.

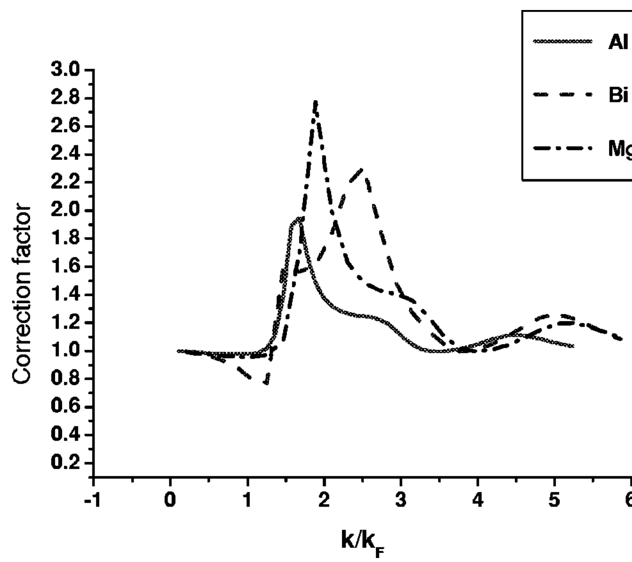

FIG. 2. (Color online) Correction factor=$\Psi^{ie}[k^2/4\pi Ze^2 \cos(kR_c)]$ at T_e=20 eV and normal density, for Bi $(Z=5,r_s=2.25,R_c=1.15a_0)$, A1 $(Z=3,r_s=2.07,R_c=1.53a_0)$, Mg $(Z=2,r_s=2.66,R_c=1.31a_0)$.

VII. SUMMARY AND DISCUSSION

The main result of the present work is in the energy balance equation (27) [with relations (28) and (29). In this equation the energy transferred from the radiation to the electrons, $\wp^{Ee}$, and the energy transferred between the electrons and ions, $\wp^{ei}-\wp^{ie}$, are written in terms of the overlap of electron and ion spectra and the effective interaction, Ψ^{ie}, which accounts for electron-ion correlations. This correlation as well as electron-electron and ion-ion correlations are accounted for also by the dynamic effective interaction [Eq. (17)] which affects the response functions $\chi(\vec{k},\omega)$ and the dynamic structure factors $S(\vec{k},\omega)$ (via the fluctuation-dissipation theorem). Even in the absence of correlations, the present unified derivation gives rise to interesting results, e.g., the quantitative expression for the effect of an external radiation field on the electron-ion relaxation rate.

The numerical estimates of the effective pair interaction presented in Sec. VI indicates that the electron-ion correlation has an enhancing effect on the rate of temperature relaxation, as well as on the inverse bremsstrahlung process. It should be noticed, however, that the quasilinear approximation used by us [Eqs. (14), (18), and (19)] excludes the possibility of coupled modes. These modes were claimed in Ref. [1] to serve as a bottleneck for the electron-ion energy transfer, and to reduce the rate by an order of magnitude or more, for strongly coupled plasmas (densities of liquefied metals). It would thus be of much interest to examine further approaches beyond the quasilinear approximation and the STLS ansatz.

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